CBSE 2026 EXAMINATIONS
XII MATHEMATICS (041)
Series 1PQRS ♦ Q.P. Code 65/1/1; 65/1/2; 65/1/3

Time Allowed : 180 Minutes **Max. Marks : 80**

General Instructions :

1. This Question paper contains **five sections - A, B, C, D and E**. Each section is compulsory. However, there are **internal choices** in some questions.
2. Section A has **18 MCQs** and **02 Assertion-Reason (A-R)** based questions of **1 mark** each.
   Section B has **05 questions** of **2 marks** each.
   Section C has **06 questions** of **3 marks** each.
   Section D has **04 questions** of **5 marks** each.
   Section E has **03 Case-study / Source-based / Passage-based** questions with **sub-parts** (**4 marks** each).
3. There is no overall choice. However, **internal choice** has been provided in
   - **02 Questions of Section B**
   - **03 Questions of Section C**
   - **02 Questions of Section D**
   - **03 Questions of Section E**

   You have to attempt only one of the alternatives in all such questions.

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SECTION A

This section has 20 multiple choice questions of 1 mark each.

01. If $2\cos^{-1} x = y$, then

(a) $0 \le y \le \pi$ (b) $-\pi \le y \le \pi$ (c) $0 \le y \le 2\pi$ (d) $-\pi \le y \le 0$

Sol. (c) Since $0 \le \cos^{-1} x \le \pi$ so, $0 \le 2\cos^{-1} x \le 2\pi$. That is, $0 \le y \le 2\pi$.

02. Which of the following cannot be the order of a row-matrix?

(a) 2×1 (b) 1×2 (c) 1×1 (d) $1\times n$

Sol. (a) Since a row matrix has only 1 row. So, 2×1 cannot be the order of a row-matrix.

03. Which of the following properties is/are true for two matrices of suitable orders?

(i) $(A+B)' = A'+B'$ (ii) $(A-B)' = B'-A'$

(iii) $(AB)' = A'B'$ (iv) $(kAB)' = kB'A'$ (k is a scalar)

(a) (i) only (b) (i), (ii) and (iii) (c) (i) and (ii) (d) (i) and (iv)

Sol. (d) Statements (i) and (iv) are correct.

04. If $\Delta_1 = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{vmatrix}$ and $\Delta_2 = \begin{vmatrix} 0 & 2 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 6 \end{vmatrix}$, then

(a) $\Delta_1 = 2\Delta_2$ (b) $\Delta_2 = -2\Delta_1$ (c) $\Delta_1 = \Delta_2$ (d) $\Delta_2 = -\Delta_1$

Sol. (b) On expanding both the determinants, $\Delta_1 = 1\times2\times3 = 6$, $\Delta_2 = 0-2(6-0)+0 = -12$.

Clearly, $\Delta_2 = -2\Delta_1$.

05. One of the values of x for which $\begin{vmatrix} \cos x & \sin x \\ -\cos x & \sin x \end{vmatrix} = 1$ is

(a) 0 (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

Sol. (b) On expanding the determinant in LHS, we get $\sin x \cos x - (-\sin x \cos x) = 1$

$\Rightarrow 2\sin x\cos x = 1$

$\Rightarrow \sin 2x = \sin\frac{\pi}{2}$

$\Rightarrow 2x = \frac{\pi}{2}$

$\therefore x = \frac{\pi}{4}$.

06. If A and B are skew symmetric matrices or same order, then which of the following matrices is also skew symmetric?

(a) AB (b) $AB + BA$ (c) $(A+B)^2$ (d) $A - B$

Sol. (d) Let $P = A - B$

$\Rightarrow P^T = (A-B)^T = A^T - B^T = -A - (-B) = B - A = -P$ [$\because A^T = -A,\ B^T = -B$

So, $P = A - B$ is also a skew symmetric matrix.

07. The least value of $f(x) = x^3 - 12x,\ x \in [0, 3]$ is

(a) -16 (b) -9 (c) 0 (d) 16

Sol. (a) For $f(x) = x^3 - 12x$, $f'(x) = 3x^2 - 12 = 3(x^2 - 4)$.

Put $f'(x) = 3(x^2 - 4) = 0$

$\Rightarrow x = 2 \in [0, 3]$

As $f(0) = 0^3 - 12\times 0 = 0$, $f(2) = 2^3 - 12\times 2 = -16$, $f(3) = 3^3 - 12\times 3 = -9$.

Hence, least value of $f(x)$ is -16.

08. If $\int \frac{3ax}{b^2 + c^2x^2}dx = A\log\left|b^2 + c^2x^2\right| + K$, then the value of A is

(a) $3a$ (b) $\frac{3a}{2b^2}$ (c) $\frac{3a}{b^2c^2}$ (d) $\frac{3a}{2c^2}$

Sol. (d) On differentiating w.r.t. x both sides, we get $\frac{3ax}{b^2 + c^2x^2} = A \times \frac{2c^2 x}{b^2 + c^2x^2}$

On comparing both the sides, we get $3a = A \times 2c^2$

$\therefore A = \frac{3a}{2c^2}$.

09. The value of $\int_{-1}^{1} \frac{x^3}{x^2 + 2|x| + 1}dx$ is

(a) 0 (b) $\log 2$ (c) $2\log 2$ (d) $\frac{1}{2}\log 2$

Sol. (a) Let $f(x) = \frac{x^3}{x^2 + 2|x| + 1}$.

As $f(-x) = \frac{(-x)^3}{(-x)^2 + 2|-x| + 1} = -\frac{x^3}{x^2 + 2|x| + 1} = -f(x)$.

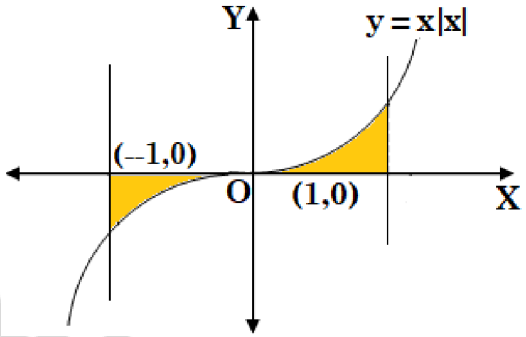
That is, $f(x)$ is an odd function.

$\therefore \int_{-1}^{1} \frac{x^3}{x^2 + 2|x| + 1}dx = 0$.

10. The area bounded by the curve $y = x|x|$, x-axis and the ordinates $x = -1$ and $x = 1$ is given by

(a) 0 (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{4}{3}$

Sol. (c) We have the curve $y = x|x| = \begin{cases} x^2, & x \geq 0 \\ -x^2, & x < 0 \end{cases}$.



$$\text{Required area} = \left|\int_{-1}^{0} -x^2 dx\right| + \int_{0}^{1} x^2 dx$$

$$= \left|-\frac{1}{3}\left[x^3\right]_{-1}^{0}\right| + \frac{1}{3}\left[x^3\right]_{0}^{1}$$

$$= \left|-\frac{1}{3}[0-(-1)]\right| + \frac{1}{3}[1-0]$$

$$= \left|-\frac{1}{3}\right| + \frac{1}{3}$$

$$= \frac{2}{3} \text{ Sq. units}.$$

11. The integrating factor of differential equation $R\frac{dx}{dy} + Px = Q$ where P, Q, R are functions of y is

(a) $e^{\int \frac{P}{Q} dy}$ (b) $e^{\int P dy}$ (c) $e^{\int \frac{P}{R} dy}$ (d) $e^{\int \frac{P}{R} dx}$

Sol. (c) Rewriting the D.E., $\frac{dx}{dy} + \frac{P}{R}x = \frac{Q}{R}$.

Clearly, the I.F. $= e^{\int \frac{P}{R} dy}$.

12. The order and degree of the differential equation $\frac{d}{dx}(e^y) = 0$ respectively are

(a) 0, 1 (b) 1, 1 (c) 2, 1 (d) 1, not defined

Sol. (b) $\frac{d}{dx}(e^y) = 0$ can be rewritten as $e^y \times \frac{dy}{dx} = 0$. Hence, order of D.E. is 1 and degree is also 1.

13. The value of p for which $\hat{i} + 2\hat{j} + 3\hat{k}$ and $2\hat{i} - p\hat{j} + \hat{k}$ are perpendicular to each other is

(a) 0 (b) 1 (c) $\frac{5}{2}$ (d) $-\frac{5}{2}$

Sol. (c) Taking the dot product of vectors 0, since the vectors are perpendicular, we get

$(\hat{i} + 2\hat{j} + 3\hat{k}).(2\hat{i} - p\hat{j} + \hat{k}) = 0$

$\Rightarrow 2 - 2p + 3 = 0$

$\therefore p = \frac{5}{2}$.

14. The value of m for which the points with position vectors $-\hat{i} - \hat{j} + 2\hat{k}$, $2\hat{i} + m\hat{j} + 5\hat{k}$ and $3\hat{i} + 11\hat{j} + 6\hat{k}$ are collinear, is

(a) 8 (b) –8 (c) 2 (d) $\frac{5}{2}$

Sol. (a) Let $A(-\hat{i} - \hat{j} + 2\hat{k})$, $B(2\hat{i} + m\hat{j} + 5\hat{k})$ and $C(3\hat{i} + 11\hat{j} + 6\hat{k})$.

Then $\overrightarrow{AB} = 3\hat{i} + (m+1)\hat{j} + 3\hat{k}$, $\overrightarrow{AC} = 4\hat{i} + 12\hat{j} + 4\hat{k}$.

As the given points A, B and C are collinear, so the d.r.'s of $\overrightarrow{AB}$ and $\overrightarrow{AC}$ must be proportional.

That is, $\frac{3}{4}=\frac{m+1}{12}=\frac{3}{4}$.

Consider $\frac{m+1}{12}=\frac{3}{4} \Rightarrow m+1=12\times\frac{3}{4}=9$

$\therefore m=8$.

15. If $|\vec{a}|=8$, $|\vec{b}|=3$ and $|\vec{a}\times\vec{b}|=12$, then the value of $|\vec{a}.\vec{b}|=$

(a) $6\sqrt{3}$ (b) $8\sqrt{3}$ (c) $12\sqrt{3}$ (d) $3\sqrt{12}$

Sol. (c) Using $|\vec{a}\times\vec{b}|^2+(\vec{a}.\vec{b})^2=|\vec{a}|^2|\vec{b}|^2$, we get $12^2+(\vec{a}.\vec{b})^2=8^2\times3^2$

$\Rightarrow(\vec{a}.\vec{b})^2=64\times9-144=9(64-16)=9\times48$

$\Rightarrow(\vec{a}.\vec{b})^2=\sqrt{9\times48}$

$\therefore|\vec{a}.\vec{b}|=3\times4\sqrt{3}=12\sqrt{3}$.

16. The length of perpendicular drawn from (2, 5, 7) on line $\frac{x}{1}=\frac{y}{0}=\frac{z}{0}$ is

(a) 0 (b) 5 (c) $\sqrt{74}$ (d) $\sqrt{78}$

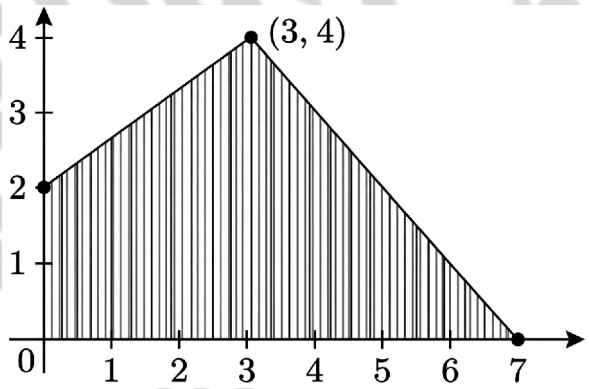
Sol. (c) In 3-D geometry, a line through (0, 0, 0) and with direction ratios 1, 0, 0 is simply the x-axis.

That is, $\frac{x}{1}=\frac{y}{0}=\frac{z}{0}$ represents the x-axis.

Now foot of perpendicular drawn from (2, 5, 7) to the x-axis is (2, 0, 0).
Therefore, we just need to find the distance between these two points, using distance formula.

Hence, required length of perpendicular $=\sqrt{0^2+5^2+7^2}=\sqrt{25+49}=\sqrt{74}$.

17. The feasible region of a linear programming problem with objective function $Z=5x+7y$ is shown below.



The maximum value of Z – minimum value of Z is

(a) 8 (b) 29 (c) 35 (d) 43

Sol. (d) Corner points are (0, 0), (0, 2), (3, 4) and (7, 0).
Now $Z_{(0,0)}=0$, $Z_{(0,2)}=14$, $Z_{(3,4)}=43$, $Z_{(7,0)}=35$.

Hence, the maximum value of Z – minimum value of $Z=43-0=43$.

18. The degree of an objective function of a linear programming problem is

(a) 0 (b) 1 (c) 2 (d) Any natural number

Sol. (b) 1. Since $Z=ax+by+c$ is the general form for an objective function in LPP.

Direction : *Question numbers* ***19*** *and* ***20*** *are Assertion (A) and Reason (R) based questions carrying* ***1 mark*** *each. Two statements are given, one labeled Assertion (A) and other labeled Reason (R). Select the correct answer from the options given below.*

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(b) Both Assertion (A) and Reason (R) are true and Reason (R) is **not** the correct explanation of Assertion (A).
(c) Assertion (A) is true but Reason (R) is false.
(d) Assertion (A) is false but Reason (R) is true.

19. **Assertion (A) :** In an experiment of throwing an unbiased die, the probability of getting a prime number given that number appearing on the die being odd is $\frac{2}{3}$.

Reason (R) : For any two events A and B, $P(A \mid B) = \frac{P(A \cup B)}{P(B)}$.

Sol. (c) Here $S = \{1,2,3,4,5,6\}$. Let $E = \{2,3,5\}$, $F = \{1,3,5\}$ $\therefore E \cap F = \{3,5\}$.

That gives, $P(E \mid F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{2}{6}}{\frac{3}{6}} = \frac{2}{3}$.

So, Assertion (A) is true but Reason (R) is false.

20. **Assertion (A) :** Lines given by $x = py + q$, $z = ry + s$ and $x = p'y + q'$, $z = r'y + s'$ are perpendicular to each other when $pp' + rr' = 1$.

Reason (R) : Two lines $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$ are perpendicular to each other if $\vec{b}_1.\vec{b}_2 = 0$.

Sol. (d) Rewriting the given lines, we get $\frac{x-q}{p} = \frac{y}{1} = \frac{z-s}{r}$ and $\frac{x-q'}{p'} = \frac{y}{1} = \frac{z-s'}{r'}$.

Clearly, their d.r.'s are $p,1,r$ and $p',1,r'$ respectively.

If the lines are perpendicular, then $p \times p' + 1 \times 1 + r \times r' = 0$ [$\because a_1a_2 + b_1b_2 + c_1c_2 = 0$ for $\perp^{er}$ lines

$\Rightarrow pp' + rr' = -1$.

So, Assertion (A) is false but Reason (R) is true.

SECTION B

This section has 5 Very Short Answer questions of 2 marks each.

21. (a) Check whether function $f(x)$ defined as $f(x) = \begin{cases} \frac{|x-3|}{2(x-3)}, & x < 3 \\ \frac{x-6}{6}, & x \geq 3 \end{cases}$ is continuous at $x = 3$ or not.

OR

(b) If $\sqrt{3}(x^2 + y^2) = 4xy$, then find $\frac{dy}{dx}$ at $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$.

Sol. (a) Redefining the function, we get $f(x) = \begin{cases} \frac{-(x-3)}{2(x-3)}, & x < 3 \\ \frac{x-6}{6}, & x \geq 3 \end{cases}$ i.e., $f(x) = \begin{cases} \frac{-1}{2}, & x < 3 \\ \frac{x-6}{6}, & x \geq 3 \end{cases}$.

Now $f(3) = \frac{3-6}{6} = \frac{-3}{6} = \frac{-1}{2}$.

LHL : $\lim\limits_{x \to 3^-} f(x) = \lim\limits_{x \to 3^-} \left(\frac{-1}{2}\right) = \frac{-1}{2}$,

RHL : $\lim\limits_{x\to 3^+} f(x) = \lim\limits_{x\to 3^+}\left(\dfrac{x-6}{6}\right) = \dfrac{3-6}{6} = \dfrac{-3}{6} = \dfrac{-1}{2}$.

Since $LHL = RHL = f(3)$, so the function $f(x)$ is continuous at $x = 3$.

OR

(b) Given $\sqrt{3}(x^2+y^2) = 4xy$

On differentiating w.r.t. x, we get $\sqrt{3}\left(2x + 2y\dfrac{dy}{dx}\right) = 4\left(x\dfrac{dy}{dx} + y\right)$

$\Rightarrow x\sqrt{3} + y\sqrt{3}\dfrac{dy}{dx} = 2x\dfrac{dy}{dx} + 2y$

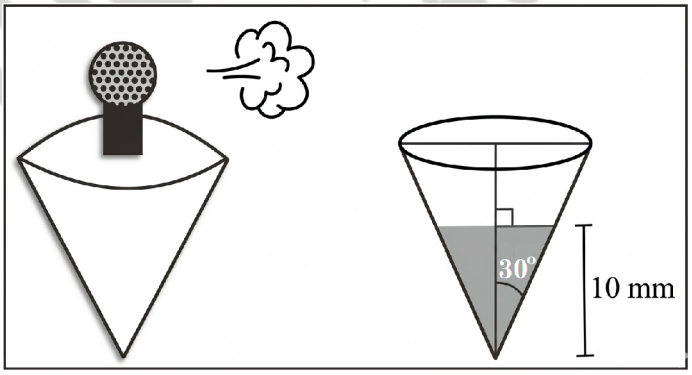
$\Rightarrow (y\sqrt{3} - 2x)\dfrac{dy}{dx} = 2y - x\sqrt{3}$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{2y - x\sqrt{3}}{y\sqrt{3} - 2x}$

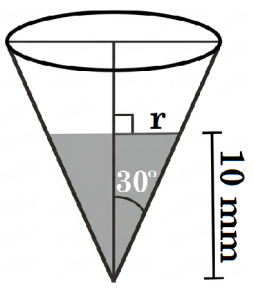
Hence, $\dfrac{dy}{dx}$ at $\left(\dfrac{1}{2}, \dfrac{\sqrt{3}}{2}\right) = \dfrac{2\times\frac{\sqrt{3}}{2} - \sqrt{3}\times\frac{1}{2}}{\sqrt{3}\times\frac{\sqrt{3}}{2} - 2\times\frac{1}{2}} = \dfrac{\sqrt{3} - \frac{\sqrt{3}}{2}}{\frac{3}{2} - 1} = \dfrac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$.

REMARK Since the question given in (b) does not ask to find $\dfrac{dy}{dx}$ specifically, so we can skip finding it in the simplified form and replace the values of x and y from the point $\left(\dfrac{1}{2}, \dfrac{\sqrt{3}}{2}\right)$ after the second step itself.

22. A room freshener bottle in the shape of an inverted cone sprays the perfume at regular intervals such that volume of the perfume in the bottle decreases at the steady rate of 1 mm^3/min. Find the rate at which level of perfume is dropping at an instant when level of perfume in the bottle is 10 mm, if the semi-vertical angle of conical bottle is $\dfrac{\pi}{6}$.



Sol. Let V be the volume of the perfume, h be the height (level) of the perfume, and r be the radius of the perfume's surface at any time t.



Given $\dfrac{dV}{dt} = -1\ mm^3/min$.

From the triangle made in the cone, we can relate r and h using the semi-vertical angle $\theta = 30^\circ$.

That is, $\tan 30^\circ = \dfrac{r}{h} \quad \Rightarrow r = \dfrac{h}{\sqrt{3}}$

Now, the volume of cone is $V=\frac{\pi}{3}r^2h=\frac{\pi}{3}\left(\frac{h}{\sqrt{3}}\right)^2 h=\frac{\pi}{9}\times h^3$

$\Rightarrow \frac{dV}{dt}=\frac{\pi}{9}\times 3h^2\times\frac{dh}{dt}$

$\Rightarrow -1=\frac{\pi}{3}\times h^2\times\frac{dh}{dt}$

$\Rightarrow \left(\frac{dh}{dt}\right)_{\text{at h=10 mm}}=-\frac{3}{\pi(10)^2}=-\frac{3}{100\pi}$ mm/min

Hence, the level of perfume is dropping at a rate of $\frac{3}{100\pi}$ mm/min .

REMARK The semi-vertical angle 30° shown in the diagram was not clearly visible, as claimed by some of the students. However, the statement in the question also mentioned the angle to be $\frac{\pi}{6}$.

23. Find the vector of magnitude 14 in the direction of $\overrightarrow{QP}$, where P and Q are the points (1, 3, 2) and (–1, 0, 8) respectively.

Sol. Here $\overrightarrow{QP}=(1-(-1))\hat{i}+(3-0)\hat{j}+(2-8)\hat{k}=2\hat{i}+3\hat{j}-6\hat{k}=\vec{r}$ *say*.

Required vector $=14\hat{r}=14\left(\frac{2\hat{i}+3\hat{j}-6\hat{k}}{\sqrt{4+9+36}}\right)=14\left(\frac{2\hat{i}+3\hat{j}-6\hat{k}}{7}\right)=2(2\hat{i}+3\hat{j}-6\hat{k})=4\hat{i}+6\hat{j}-12\hat{k}$.

24. Vectors $\vec{a}=3\hat{i}-2\hat{j}+2\hat{k}$ and $\vec{b}=\hat{i}+2\hat{k}$ represent the two adjacent sides of a parallelogram. Find the vectors representing its diagonals and hence find their lengths.

Sol. The vectors representing the diagonals are $\vec{d}_1=\vec{a}+\vec{b}=4\hat{i}-2\hat{j}+4\hat{k}$, $\vec{d}_2=\pm(\vec{a}-\vec{b})=\pm(2\hat{i}-2\hat{j})$.

The lengths of diagonals are given by the magnitudes of diagonal vectors $\vec{d}_1$ and $\vec{d}_2$.

Hence, $\left|\vec{d}_1\right|=\sqrt{4^2+(-2)^2+4^2}=\sqrt{36}=6$ units and $\left|\vec{d}_2\right|=\sqrt{(\pm2)^2+(\mp2)^2+0^2}=\sqrt{8}=2\sqrt{2}$ units.

25. (a) Simplify : $\tan^{-1}\left(\frac{\cos 2x-\sin 2x}{\cos 2x+\sin 2x}\right)$, $0<x<\frac{\pi}{4}$.

OR

(b) Evaluate : $\tan\left(\sin^{-1}1-\cos^{-1}\left(-\frac{1}{2}\right)\right)$.

Sol. (a) Let $y=\tan^{-1}\left(\frac{\cos 2x-\sin 2x}{\cos 2x+\sin 2x}\right)$, $0<x<\frac{\pi}{4}$

$\Rightarrow y=\tan^{-1}\left(\frac{1-\tan 2x}{1+1.\tan 2x}\right)=\tan^{-1}\left(\frac{\tan\frac{\pi}{4}-\tan 2x}{1+\tan\frac{\pi}{4}.\tan 2x}\right)=\tan^{-1}\left[\tan\left(\frac{\pi}{4}-2x\right)\right]$

$\therefore y=\left(\frac{\pi}{4}-2x\right)$

$\because 0<x<\frac{\pi}{4} \Rightarrow 0<2x<\frac{\pi}{2} \Rightarrow -\frac{\pi}{4}<2x-\frac{\pi}{4}<\frac{\pi}{4} \Rightarrow -\frac{\pi}{4}<\frac{\pi}{4}-2x<\frac{\pi}{4}$.

Also $\tan^{-1}\tan x=x$, when $x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$.

OR

(b) $\tan\left(\sin^{-1}1-\cos^{-1}\left(-\frac{1}{2}\right)\right)=\tan\left(\frac{\pi}{2}-\frac{2\pi}{3}\right)=\tan\left(-\frac{\pi}{6}\right)=-\tan\frac{\pi}{6}=-\frac{1}{\sqrt{3}}$.

SECTION C

There are 6 Short Answer questions in this section. Each is of 3 marks.

26. Evaluate $\int_0^1 x\tan^{-1}x\,dx$.

Sol. Consider $\int x\tan^{-1}x\,dx=\tan^{-1}x\int x\,dx-\int\left[\frac{d}{dx}(\tan^{-1}x)\int x\,dx\right]dx$

$$=\frac{x^2}{2}\tan^{-1}x-\frac{1}{2}\int\frac{x^2}{1+x^2}dx$$
$$=\frac{x^2}{2}\tan^{-1}x-\frac{1}{2}\int\left(1-\frac{1}{1+x^2}\right)dx$$
$$=\frac{x^2}{2}\tan^{-1}x-\frac{1}{2}\left(x-\tan^{-1}x\right)$$

Now $\int_0^1 x\tan^{-1}x\,dx=\left[\frac{x^2}{2}\tan^{-1}x-\frac{1}{2}\left(x-\tan^{-1}x\right)\right]_0^1$

$$=\left[\frac{1}{2}\tan^{-1}1-\frac{1}{2}\left(1-\tan^{-1}1\right)\right]-\left[\frac{0}{2}\tan^{-1}0-\frac{1}{2}\left(0-\tan^{-1}0\right)\right]$$
$$=\frac{1}{2}\times\frac{\pi}{4}-\frac{1}{2}+\frac{1}{2}\times\frac{\pi}{4}$$
$$=\frac{\pi}{4}-\frac{1}{2}.$$

27. (a) Find $\int\sqrt{\frac{x+2}{x-2}}dx$.

OR

(b) Evaluate $\int\frac{x^2}{(x^2+9)(x^2+16)}dx$.

Sol. (a) Let $I=\int\sqrt{\frac{x+2}{x-2}}dx$

$$\Rightarrow I=\int\sqrt{\frac{x+2}{x-2}\times\frac{x+2}{x+2}}dx$$
$$\Rightarrow I=\int\frac{x+2}{\sqrt{x^2-4}}dx$$
$$\Rightarrow I=\frac{1}{2}\int\frac{2x}{\sqrt{x^2-4}}dx+\int\frac{2}{\sqrt{x^2-4}}dx=\frac{1}{2}\times2\sqrt{x^2-4}+2\log\left|x+\sqrt{x^2-4}\right|+C$$
$$\therefore I=\sqrt{x^2-4}+2\log\left|x+\sqrt{x^2-4}\right|+C \qquad \left[\because\int\frac{f'(x)}{\sqrt{f(x)}}dx=2\sqrt{f(x)}+C\right.$$

OR

(b) Consider $\frac{x^2}{(x^2+9)(x^2+16)}=\frac{t}{(t+9)(t+16)}=\frac{A}{t+9}+\frac{B}{t+16}$, where $t=x^2$

$\Rightarrow t = A(t+16) + B(t+9)$

On comparing the like terms on both the sides, we get $A = -\frac{9}{7}$, $B = \frac{16}{7}$.

Therefore, $\int \frac{x^2}{(x^2+9)(x^2+16)}dx = \int\left[\frac{16}{7(x^2+16)} - \frac{9}{7(x^2+9)}\right]dx$

$$= \frac{16}{7}\times\frac{1}{4}\tan^{-1}\frac{x}{4} - \frac{9}{7}\times\frac{1}{3}\tan^{-1}\frac{x}{3} + C$$

$$= \frac{4}{7}\tan^{-1}\frac{x}{4} - \frac{3}{7}\tan^{-1}\frac{x}{3} + C.$$

28. If $I_1 = \int_{-\pi/4}^{\pi/4} \frac{dx}{1+\cos 2x}$ and $I_2 = \int_{-1/2}^{1/2} |x|dx$, then show that $I_1 - 4I_2 = 0$.

Sol. Let $f(x) = \frac{1}{1+\cos 2x}$ and $g(x) = |x|$.

$\because f(-x) = \frac{1}{1+\cos 2(-x)} = \frac{1}{1+\cos 2x} = f(x)$ and $g(-x) = |-x| = |x| = g(x)$.

So, both the functions f and g are even.

Therefore, $I_1 = \int_{-\pi/4}^{\pi/4} \frac{dx}{1+\cos 2x} = 2\int_0^{\pi/4} \frac{dx}{1+\cos 2x} = 2\int_0^{\pi/4} \frac{dx}{2\cos^2 x} = \int_0^{\pi/4} \sec^2 x\, dx$

$\Rightarrow I_1 = [\tan x]_0^{\pi/4} = \tan\frac{\pi}{4} - \tan 0 = 1 - 0 = 1$ ...(i)

Also $I_2 = \int_{-1/2}^{1/2} |x|dx = 2\int_0^{1/2} |x|dx = 2\int_0^{1/2} x\,dx = 2\left[\frac{x^2}{2}\right]_0^{1/2} = \left[\frac{1}{4} - 0\right] = \frac{1}{4}$

$\Rightarrow 4I_2 = 1$ ...(ii)

By (i) and (ii), we get $I_1 - 4I_2 = 1 - 1 = 0$.

29. (a) Find the general solution of the following differential equation $x^2\frac{dy}{dx} = x^2 + xy + y^2$.

OR

(b) Find the particular solution of the differential equation $xy\frac{dy}{dx} = (x+2)(y+2)$, given that $y(1) = -1$.

Sol. (a) Rewriting the D.E., we get $\frac{dy}{dx} = 1 + \frac{y}{x} + \frac{y^2}{x^2}$

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

So, $v + x\frac{dv}{dx} = 1 + v + v^2$

$\Rightarrow x\frac{dv}{dx} = 1 + v^2$

$\Rightarrow \int \frac{dv}{1+v^2} = \int \frac{dx}{x}$

$\Rightarrow \tan^{-1} v = \log|x| + C$

$\Rightarrow \tan^{-1}\frac{y}{x} = \log|x| + C$ or $y = x\tan\left[\log|x| + C\right]$.

OR

(b) Rewriting the D.E., we get $\frac{y\,dy}{y+2} = \frac{(x+2)\,dx}{x}$

$\Rightarrow \int \frac{(y+2)-2}{y+2}dy = \int\left(\frac{x}{x} + \frac{2}{x}\right)dx$

$\Rightarrow \int\left(1 - \frac{2}{y+2}\right)dy = \int\left(1 + \frac{2}{x}\right)dx$

$\Rightarrow y - 2\log|y+2| = x + 2\log|x| + C$

It is given that $y(1) = -1$. So, $-1 - 2\log|-1+2| = 1 + 2\log|1| + C$

$\Rightarrow C = -2$ $[\because \log 1 = 0$

Hence, the required particular solution is $y - 2\log|y+2| = x + 2\log|x| - 2$.

30. Solve the following linear programming problem graphically.

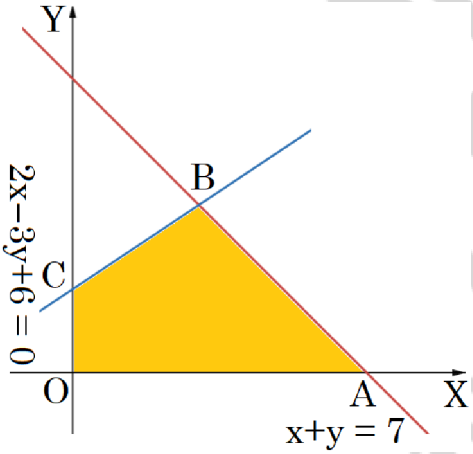
Minimize $Z = 13x - 15y$

Subject to constraints $x + y \le 7$, $2x - 3y + 6 \ge 0$, $x \ge 0, y \ge 0$.

Sol. Consider the graph shown.

The corner points are O(0, 0), A(7, 0), B(3, 4) and C(0, 2).

Therefore, $Z_O = 0$, $Z_A = 91$, $Z_B = -21$, $Z_C = -30$.



Hence, minimum value of Z is –30, which occurs at the point (0, 2).

31. (a) Out of two bags, bag I contains 3 red and 4 white balls and bag II contains 8 red and 6 white balls. A die is thrown. If it shows a number less than 3 then a ball is drawn at random from bag I, otherwise a ball is drawn at random from bag II. Find the probability that the ball drawn from one of the bags is a red ball.

OR

(b) The probability of simultaneous occurrence of at least one of the two events X and Y is a. If the probability that exactly one of the events X, Y occurs is b, prove that $P(X') + P(Y') = 2 - 2a + b$.

Sol. (a) We have Bag I contains 3 red and 4 white balls (Total = 7); Bag II contains 8 red and 6 white balls (Total = 14).

Since there will be 6 outcomes when a die is thrown i.e., {1, 2, 3, 4, 5, 6}.

- Bag I is chosen if the number is less than 3, i.e., {1, 2} $\therefore$ P(Bag I) $= \frac{2}{6} = \frac{1}{3}$

- Bag II is chosen if the number is 3 or more, i.e., {3, 4, 5, 6} $\therefore$ P(Bag II) $= \frac{4}{6} = \frac{2}{3}$

Also $P(\text{Red} \mid \text{Bag I}) = \frac{3}{7}$, $P(\text{Red} \mid \text{Bag II}) = \frac{8}{14} = \frac{4}{7}$

Therefore, $P(\text{Red}) = P(\text{Bag I}).P(\text{Red} \mid \text{Bag I}) + P(\text{Bag II}).P(\text{Red} \mid \text{Bag II})$

$\Rightarrow P(\text{Red}) = \frac{1}{3}.\frac{3}{7} + \frac{2}{3}.\frac{4}{7} = \frac{3}{21} + \frac{8}{21}$

$\therefore P(\text{Red}) = \frac{11}{21}$.

OR

(b) We have $P(X \cup Y) = a$.

Also $P(X \cap Y') + P(X' \cap Y) = b$

$\Rightarrow P(X) - P(X \cap Y) + P(Y) - P(X \cap Y) = b$

$\Rightarrow [P(X) + P(Y) - P(X \cap Y)] - P(X \cap Y) = b$

$\Rightarrow P(X \cup Y) - P(X \cap Y) = b$

$\Rightarrow P(X \cap Y) = a - b$

Also $P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$

$\Rightarrow a = P(X) + P(Y) - (a - b)$

$\Rightarrow 2a - b = P(X) + P(Y)$

$\Rightarrow -P(X) - P(Y) = b - 2a$

$\Rightarrow [1 - P(X)] + [1 - P(Y)] = 2 + b - 2a$

$\therefore P(X') + P(Y') = 2 - 2a + b$.

SECTION D

There are 4 Long Answer questions in this section. Each is of 5 marks.

32. (a) A relation R is defined on Z, the set of integers, as $R = \{(x, y) : |x - y|$ is divisible by a prime number 'p', $x, y \in Z\}$. Check whether R is an equivalence relation or not.

OR

(b) A function $f : \mathbb{R} - \left\{\frac{3}{5}\right\} \to \mathbb{R} - \left\{\frac{3}{5}\right\}$ is defined as $f(x) = \frac{3x + 2}{5x - 3}$. Show that f is one-one and onto.

Sol. (a) For any $x \in Z$, $|x - x| = 0$.

Since 0 is divisible by any prime number p, so $(x, x) \in R$. Thus, R is reflexive.

Let $(x, y) \in R$. This implies, $|x - y|$ is divisible by a prime number 'p'.

Since $|x - y| = |-(y - x)| = |y - x|$, it follows that $|y - x|$ is also divisible by p.

That means, $(y, x) \in R$. Thus, R is symmetric.

Let $(x, y) \in R$ and $(y, z) \in R$.

- $|x - y|$ is divisible by p $\Rightarrow x - y = \pm k\,p$, where $k \in Z$
- $|y - z|$ is divisible by p $\Rightarrow y - z = \pm m\,p$, where $m \in Z$

Adding the expressions, we get $(x - y) + (y - z) = x - z = \pm(k + m)\,p$

That is, $|x - z|$ is divisible by p. Therefore, $(x, z) \in R$.

Thus, R is transitive.

Since R is reflexive, symmetric, and transitive, it is an equivalence relation.

OR

(b) Let $f(x_1) = f(x_2)$ for x_1, x_2 in the domain.

That is, $\frac{3x_1+2}{5x_1-3} = \frac{3x_2+2}{5x_2-3}$

$\Rightarrow 15x_1x_2 - 9x_1 + 10x_2 - 6 = 15x_1x_2 + 10x_1 - 9x_2 - 6$

$\Rightarrow -9x_1 + 10x_2 = 10x_1 - 9x_2$

$\Rightarrow -19x_1 = -19x_2$

$\Rightarrow x_1 = x_2$

$\therefore$ f is one-one.

Let y be an element in the codomain $\mathbb{R} - \left\{\frac{3}{5}\right\}$.

We take $y = f(x) = \frac{3x+2}{5x-3}$

$\Rightarrow 5xy - 3y = 3x + 2$

$\Rightarrow 5xy - 3x = 2 + 3y$

$\Rightarrow x(5y-3) = 2 + 3y$

$\Rightarrow x = \frac{2+3y}{5y-3}$

For any $y \in \mathbb{R} - \left\{\frac{3}{5}\right\}$, the term in denominator i.e., $5y - 3 \neq 0$.

This, x is always defined and belongs to the domain $\mathbb{R} - \left\{\frac{3}{5}\right\}$.

Since every y in the codomain has a pre-image x in the domain, f is onto.

33. (a) If $A = \begin{bmatrix} 0 & 2 & 1 \\ -2 & -1 & -2 \\ 1 & -1 & 0 \end{bmatrix}$, find A^{-1} and use it to solve the following system of equations :

$-2y + z = 7,\ 2x - y - z = 8,\ x - 2y = 10.$

OR

(b) If $\begin{bmatrix} 3 & -1 & \sin 3x \\ -7 & 4 & \cos 2x \\ -11 & 7 & 2 \end{bmatrix}$ is a singular matrix, then find all values of x, where $x \in \left[0, \frac{\pi}{2}\right]$.

Sol. (a) For $A = \begin{bmatrix} 0 & 2 & 1 \\ -2 & -1 & -2 \\ 1 & -1 & 0 \end{bmatrix}$, $|A| = 0 - 2(0+2) + 1(2+1) = -4 + 3 = -1 \neq 0$ $\therefore A^{-1}$ exists.

For the adj.A, we shall write a matrix of cofactors of elements a_{ij}, $\text{adj.A} = \begin{bmatrix} -2 & -1 & -3 \\ -2 & -1 & -2 \\ 3 & 2 & 4 \end{bmatrix}$.

$\therefore A^{-1} = \frac{\text{adj.A}}{|A|} = \begin{bmatrix} 2 & 1 & 3 \\ 2 & 1 & 2 \\ -3 & -2 & -4 \end{bmatrix}$

Consider the system of equations $-2y+z=7,\ 2x-y-z=8,\ x-2y=10$.

Let $M=\begin{bmatrix}0 & -2 & 1\\ 2 & -1 & -1\\ 1 & -2 & 0\end{bmatrix}=A^T,\ X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}$ and $B=\begin{bmatrix}7\\ 8\\ 10\end{bmatrix}$.

Clearly, $MX=B$

$\Rightarrow X=M^{-1}B$

$\Rightarrow X=\begin{bmatrix}2 & 2 & -3\\ 1 & 1 & -2\\ 3 & 2 & -4\end{bmatrix}\begin{bmatrix}7\\ 8\\ 10\end{bmatrix}$ $[\because M=A^T\Rightarrow M^{-1}=(A^{-1})^T$

$\Rightarrow X=\begin{bmatrix}14+16-30\\ 7+8-20\\ 21+16-40\end{bmatrix}$

$\Rightarrow\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}0\\ -5\\ -3\end{bmatrix}$

$\therefore x=0,\ y=-5,\ z=-3$.

OR

(b) Recall that, a matrix is singular if its determinant is zero (0).

That is, $\begin{vmatrix}3 & -1 & \sin 3x\\ -7 & 4 & \cos 2x\\ -11 & 7 & 2\end{vmatrix}=0$

On expanding, we get $3(8-7\cos 2x)-(-1)(-14+11\cos 2x)+\sin 3x(-49+44)=0$

$\Rightarrow 24-21\cos 2x-14+11\cos 2x-5\sin 3x=0$

$\Rightarrow 10-10\cos 2x-5\sin 3x=0$

$\Rightarrow 2-2\cos 2x-\sin 3x=0$

$\Rightarrow 2-2(1-2\sin^2 x)-(3\sin x-4\sin^3 x)=0$

$\Rightarrow 2-2+4\sin^2 x-3\sin x+4\sin^3 x=0$

$\Rightarrow \sin x(4\sin^2 x+4\sin x-3)=0$

$\Rightarrow \sin x(2\sin x-1)(2\sin x+3)=0$

Either $\sin x=0$ or $(2\sin x-1)=0$ or $(2\sin x+3)=0$

$\Rightarrow x=0$ or $\sin x=\frac{1}{2}$ or $\sin x=-\frac{3}{2}$

But $\sin x=-\frac{3}{2}$ is not possible.

Hence, the possible values of x are $0,\frac{\pi}{6}$ such that $x\in\left[0,\frac{\pi}{2}\right]$.

34. If $x=\cos t,\ y=\cos mt$, prove that $(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}+m^2y=0$.

Sol. We have $x=\cos t,\ y=\cos mt$

$\Rightarrow \frac{dx}{dt}=-\sin t,\ \frac{dy}{dt}=-m\sin mt$

As $\frac{dy}{dx}=\frac{dy}{dt}\times\frac{dt}{dx}$ so, $\frac{dy}{dx}=\frac{-m\sin mt}{-\sin t}=\frac{m\sin mt}{\sin t}$

$\Rightarrow \sin t\frac{dy}{dx}=m\sin mt$

Now differentiating w.r.t. x, $\sin t\frac{d^2y}{dx^2}+\frac{dy}{dx}\times\cos t\frac{dt}{dx}=m^2\cos mt\times\frac{dt}{dx}$

$\Rightarrow \sin t\frac{d^2y}{dx^2}+\frac{dy}{dx}\times x\left(\frac{1}{-\sin t}\right)=m^2\ y\times\left(\frac{1}{-\sin t}\right)$

$\Rightarrow \sin^2 t\frac{d^2y}{dx^2}-\frac{dy}{dx}\times x=-m^2\ y$

$\Rightarrow (1-\cos^2 t)\frac{d^2y}{dx^2}-x\frac{dy}{dx}+m^2\ y=0$

Thus, $(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}+m^2y=0$.

35. Check whether the lines given by $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-4}{5}=\frac{y-1}{2}=z$ are parallel or not. If parallel, find the distance between them, otherwise find their point of intersection, if the lines are intersecting

Sol. Let $L_1:\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}=\lambda$ and $L_2:\frac{x-4}{5}=\frac{y-1}{2}=\frac{z}{1}=\mu$.

Their d.r.'s are 2, 3, 4 and 5, 2, 1 respectively.

Two lines are parallel if their direction vectors are proportional.

Since $\frac{2}{5}\neq\frac{3}{5}\neq\frac{4}{1}$ i.e., the ratios are not equal, the lines are **not parallel**.

Consider the random points on both the lines $P(2\lambda+1, 3\lambda+2, 4\lambda+3)$ and $Q(5\mu+4, 2\mu+1, \mu)$ respectively.

If the lines intersect, then P and Q must coincide i.e., there must be values for λ and μ that satisfy all three coordinates in the random points P and Q.

That is, $2\lambda+1=5\mu+4$ ...(i), $3\lambda+2=2\mu+1$ ...(ii), $4\lambda+3=\mu$ ...(iii)

Solving (i) and (ii), we get $\lambda=-1, \mu=-1$

LHS of (iii): $4\lambda+3=4(-1)+3=-1$ and RHS of (iii): $\mu=-1$.

That is, values of $\lambda=-1, \mu=-1$ satisfy (iii).

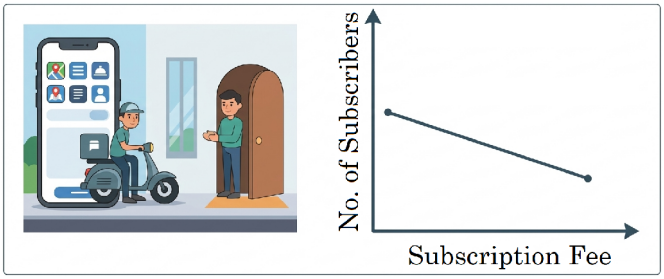
Hence, the points P and Q will coincide.

Therefore, the lines are intersecting and their point of intersection is (–1, –1, –1).

SECTION E

In this section there are 3 Case Study questions of 4 marks each.

36. An online delivery company in a city has 5000 subscribers and collects annual subscription fees of ₹300 per subscriber for unlimited free deliveries.



The company wishes to increase the annual subscription fee. It is predicted that, for every increase of ₹1, ten subscribers will discontinue. Assume that the company increased the annual fee by ₹x.

Based on the given information, answer the following questions.

(i) How many subscribers will discontinue after an increase of ₹x in the annual fee?

(ii) If R(x) denotes the total revenue collected after the increase of ₹x in subscription fee, express R(x) as a function of x.

(iii) (a) Find the value of x for which R(x) is maximum.

OR

(iii) (b) Find the sub-intervals of (0, 5000) in which R(x) is increasing and decreasing.

Sol. **(i)** Since 10 subscribers discontinue for every ₹1 increase, for an increase of ₹x, the number of discontinuing subscribers is 10x.

(ii) $R(x) = (5000 - 10x)(300 + x)$

$\Rightarrow R(x) = 1500000 + 5000x - 3000x - 10x^2$

$\therefore R(x) = -10x^2 + 2000x + 1500000$.

(iii) (a) For $R(x) = -10x^2 + 2000x + 1500000$

$\Rightarrow R'(x) = -20x + 2000$ and $R''(x) = -20$

For $R'(x) = 0, -20(x - 100) = 0 \quad \Rightarrow x = 100$

$\because R''(x = 100) = -20 < 0$

$\therefore R(x)$ is maximum when $x = ₹100$.

OR

(iii) (b) For $R(x) = -10x^2 + 2000x + 1500000$

$\Rightarrow R'(x) = -20x + 2000 = -20(x - 100)$

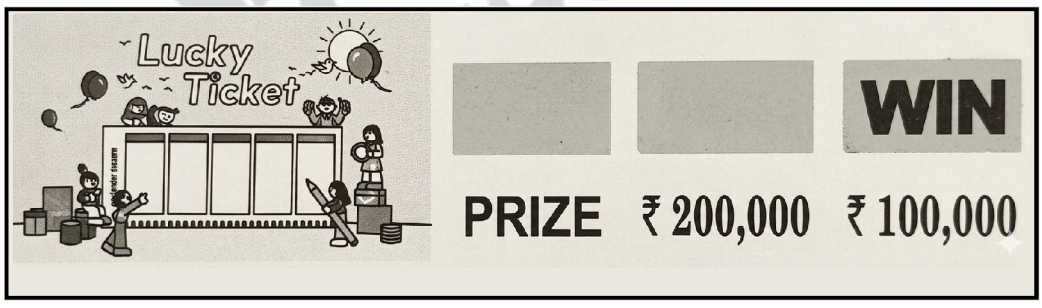
For $R'(x) = 0, -20(x - 100) = 0 \quad \Rightarrow x = 100$

When $x \in (0, 100]$, $R'(x) \geq 0$. So, R(x) increases in $x \in (0, 100]$.

When $x \in [100, 5000)$, $R'(x) \leq 0$. So, R(x) decreases in $x \in [100, 5000)$.

REMARK The interval is likely (0, 500) as 10x cannot be more than 5000 in normal cases, as it shall lead to negative revenue. However revenue can be negative in *rare* cases - if the seller doesn't have any sale but has returns of sales.

37. In an online jackpot, there is one first prize of ₹300000, two second prizes of ₹200000 each and three third prizes of ₹50000 each.



A total of 100000 jackpot tickets each costing ₹100 were sold there by raising a fund of ₹10000000.

Rohan bought one ticket.

Based on the given information, answer the following questions.

(i) What are the possible amounts, the person can win?

(ii) (a) What is the probability that the person wins at least ₹200000?

OR

(ii) (b) What is the probability that the person does not win any amount?

(iii) In another jackpot, Rohan also bought a ticket having a prize money of ₹500000. The chances of winning the jackpot are 1 in 100000. Find the probability that on exactly one of tickets he wins the jackpot.

Sol. **(i)** From the problem description, the possible winning amounts (prizes) are:

- ₹300000 (First Prize)
- ₹200000 (Second Prize)
- ₹50000 (Third Prize)
- ₹0 (If the person does not win any of the six prizes)

(ii) (a) To win at least ₹200000, the person must win either the First Prize or one of the Second Prizes.

Number of First Prizes $= 1$, Number of Second Prizes $= 2$.

So, total favorable outcomes $= 1 + 2 = 3$. Also, total tickets sold $= 100000$.

$\therefore$ Required probability $= \dfrac{3}{100000}$.

OR

(ii) (b) The total number of winning tickets is 1 (First Prize) $+ 2$ (Second Prize) $+ 3$ (Third Prize) $= 6$ tickets.

Clearly, number of non-winning tickets $100000 - 6 = 99994$.

$\therefore$ Required probability $= \dfrac{99994}{100000}$.

(iii) Rohan has two different tickets from two different jackpots.

Let A : wins Jackpot 1 (6 winning tickets out of 100000).

Also let B : wins Jackpot 2 (1 winning ticket out of 100000).

That is, $P(A) = \dfrac{6}{100000}$, $P(A') = \dfrac{99994}{100000}$ and $P(B) = \dfrac{1}{100000}$, $P(B') = \dfrac{99999}{100000}$.

To win on **exactly one** ticket, he must either win A and lose B, or lose A and win B.

$\therefore$ Required probability $= P(A) \times P(B') + P(A') \times P(B)$

$$= \frac{6}{100000} \times \frac{99999}{100000} + \frac{99994}{100000} \times \frac{1}{100000} = \frac{599994 + 99994}{10000000000}$$

$$= \frac{699988}{10000000000}.$$

REMARK The reason we must consider both the jackpots in part (iii) - and not just the second one - is because the question is asking for the probability of a very specific outcome: **"the probability that on exactly one of the tickets he wins"**.

In plain English, this means: Rohan wins exactly one of his two tickets (i.e., he wins on one ticket and loses on the other).

He has two separate tickets now.

- Ticket 1 → from the first jackpot (one prize of ₹300000, two prizes of ₹200000, three prizes of ₹50000)
- Ticket 2 → from the second jackpot (one prize of ₹500000, with probability 1 in 100,000)

"Exactly one ticket wins" refers to exactly one of these two tickets being a winner, across the two different jackpots.

If we only looked at the second jackpot, we would be answering a completely different question, such as: **"probability he wins the second jackpot"** (which is just $\dfrac{1}{100000}$). But that is not what the question is asking.

38. Roundabouts are often made on busy roads to ease the traffic and avoid red lights.



One such round-about is made such that equation representing its boundary is given by $C_1 : x^2 + y^2 = 64$.

There is circular pond with a fountain in the middle of the roundabout whose equation is given by $C_2 : x^2 + y^2 = 4$.

Based on the given information, answer the following questions.

(i) Represent the given equations C_1 and C_2 with the help of a diagram.

(ii) Express y as a function of x, ($y = f(x)$), for both C_1 and C_2.

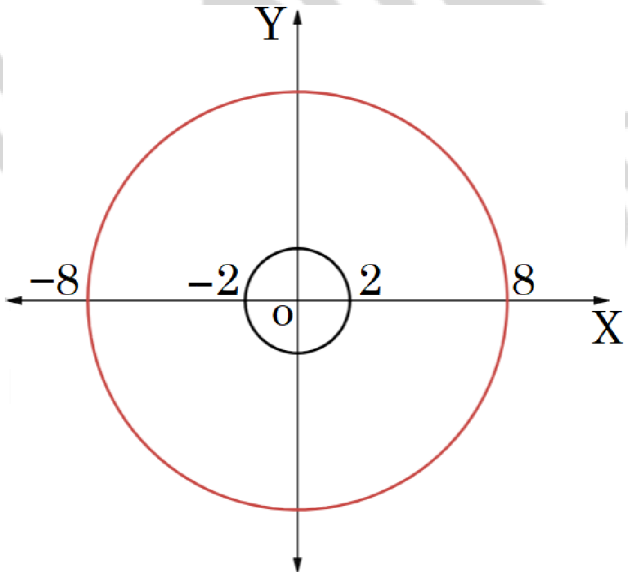
(iii) (a) Using integration, find the area of region covered by the roundabout.

OR

(iii) (b) Using integration, find the area of region covered by circular pond.

Sol. **(i)** For $C_1 : x^2 + y^2 = 64$, centre is at (0, 0) and radius is of 8 units.

For $C_2 : x^2 + y^2 = 4$, centre is at (0, 0) and radius is of 2 units.



(ii) For $C_1 : x^2 + y^2 = 64 \quad \Rightarrow y^2 = 64 - x^2 \quad \Rightarrow y = \pm\sqrt{64 - x^2}$ or $f(x) = \pm\sqrt{64 - x^2}$.

For $C_2 : x^2 + y^2 = 4 \quad \Rightarrow y^2 = 4 - x^2 \quad \Rightarrow y = \pm\sqrt{4 - x^2}$ or $f(x) = \pm\sqrt{4 - x^2}$.

(iii) (a) Required area $= 2 \times \int_{-8}^{8} \sqrt{64 - x^2}\, dx$

$$= 2\left[\frac{x}{2}\sqrt{64 - x^2} + \frac{64}{2}\sin^{-1}\frac{x}{8}\right]_{-8}^{8}$$

$$= 2\left[\left(0+32\times\frac{\pi}{2}\right)-\left(0+32\left(-\frac{\pi}{2}\right)\right)\right]$$

$$= 2\left[32\pi\right] = 64\pi \text{ Sq. units}.$$

REMARK Why you should not take the area of $C_1 - C_2$?

The roundabout is C_1, which is the boundary of the circular traffic island. Its area is just the area of the circle C_1, which is 64π Sq. units.

The question explicitly asks for the region covered by the roundabout - that's the whole big circle area. The pond C_2 is a separate feature in the middle of the roundabout, but the area of the roundabout includes the pond, since the **roundabout is the entire circular region** inside the boundary C_1. So we do not subtract C_2.

• If the question had asked to find **paved area**, then we shall get the area of $C_1 - C_2$.

Further more, if the question were "find the area of the roundabout excluding the pond", then we would take $C_1 - C_2$ area, but that's not what they asked.

OR

(iii) (b) Required area $= 4\int_0^2 \sqrt{4-x^2}\,dx$

$$= 4\left(\frac{x}{2}\sqrt{4-x^2}+\frac{4}{2}\sin^{-1}\frac{x}{2}\right)_0^2$$

$$= 4\left(\left[0+2\times\frac{\pi}{2}\right]-\left[0+0\right]\right)$$

$$= 4\pi \text{ Sq. units}.$$

☑ Q.P. Code - 65/1/2

05. The order and degree of the differential equation $\frac{d}{dx}(\sin y) = y^2$ respectively are

(a) 1, 1 (b) 2, 1 (c) 2, 2 (d) 1, 2

Sol. (a) $\because \frac{d}{dx}(\sin y) = y^2 \Rightarrow \cos y \times \frac{dy}{dx} = y^2$

$\therefore$ Order : 1 and Degree : 1.

09. The length of perpendicular drawn from the point (1, 2, 3) on line $\frac{x}{0} = \frac{y}{1} = \frac{z}{0}$ is

(a) 0 (b) 6 (c) $\sqrt{10}$ (d) $\sqrt{14}$

Sol. (c) In 3-D geometry, a line through (0, 0, 0) and with direction ratios 0, 1, 0 is simply the y-axis.

That is, $\frac{x}{0} = \frac{y}{1} = \frac{z}{0}$ represents the y-axis.

Now foot of perpendicular drawn from (1, 2, 3) to the y-axis is (0, 2, 0).

Therefore, we just need to find the distance between these two points, using distance formula.

Hence, required length of perpendicular $= \sqrt{1^2+0^2+3^2} = \sqrt{1+9} = \sqrt{10}$.

12. If $\tan^{-1} x = 3y$, then

(a) $-\frac{\pi}{2} < y < \frac{\pi}{2}$ (b) $-\frac{3\pi}{2} < y < \frac{3\pi}{2}$ (c) $-\frac{\pi}{6} < y < \frac{\pi}{6}$ (d) $-\frac{\pi}{6} \le y \le \frac{\pi}{6}$

Sol. (c) As $-\frac{\pi}{2} < \tan^{-1} x < \frac{\pi}{2} \quad \Rightarrow -\frac{\pi}{2} < 3y < \frac{\pi}{2} \quad \therefore -\frac{\pi}{6} < y < \frac{\pi}{6}$.

13. Which of the following cannot be an order of a column-matrix?

(a) 1×2 (b) 2×1 (c) 1×1 (d) $m\times1$, where $m \in N$

Sol. (a) Since a column matrix has only 1 column. So, 1×2 cannot be the order of a column-matrix.

17. If A and B are symmetric matrices of same order, then which of the following matrices is a skew-symmetric matrix?

(a) A^2+B^2 (b) A^2-B^2 (c) $AB+BA$ (d) $AB-BA$

Sol. (d) Let $P = AB-BA$

$\Rightarrow P^T = (AB-BA)^T = (AB)^T-(BA)^T = B^TA^T - A^TB^T$

$\Rightarrow P^T = BA - AB = -P$ $[\because A^T = A,\ B^T = B$

Thus, $P = AB-BA$ is a skew-symmetric matrix.

18. The absolute maximum value of $f(x) = x^2+1$ in $[-5, 2]$ is

(a) 26 (b) 1 (c) 5 (d) 2

Sol. (a) $\because f'(x) = 2x$

For $f'(x) = 0$, $2x = 0 \Rightarrow x = 0 \in [-5, 2]$.

Now $f(-5) = (-5)^2+1 = 26$, $f(0) = 0^2+1 = 1$, $f(2) = 2^2+1 = 5$.

Hence, absolute maximum value of $f(x)$ is 26.

21. A vector $\vec{a}$ of magnitude 14 has direction ratios $<2, 3, -6>$. Find the projection of the vector $\vec{a}$ on $\hat{i}$.

Sol. The d.r.'s of $\vec{a}$ are $<2, 3, -6>$.

Let $\vec{r} = 2\hat{i}+3\hat{j}-6\hat{k}$ be such that $\vec{r} \| \vec{a}$.

Then $\hat{r} = \frac{\vec{r}}{|\vec{r}|} = \frac{2\hat{i}+3\hat{j}-6\hat{k}}{\sqrt{4+9+36}} = \frac{2\hat{i}+3\hat{j}-6\hat{k}}{\sqrt{49}} = \frac{2\hat{i}+3\hat{j}-6\hat{k}}{7}$.

Thus, $\hat{r}$ is also the unit vector in the direction of $\vec{a}$.

Now $\vec{a} = 14\times$ a unit vector in the direction of $\vec{a}$.

That is, $\vec{a} = 14\times\frac{2\hat{i}+3\hat{j}-6\hat{k}}{7} = 4\hat{i}+6\hat{j}-12\hat{k}$

Hence, the projection of the vector $\vec{a}$ on $\hat{i}$ is $\vec{a}.\hat{i} = (4\hat{i}+6\hat{j}-12\hat{k}).\hat{i} = 4$.

25. (a) Simplify : $\cot^{-1}\sqrt{\frac{1+\cos 2x}{1-\cos 2x}}$, $x \in \left(0, \frac{\pi}{2}\right)$.

OR

(b) Evaluate : $\sin\left(\tan^{-1}(-\sqrt{3}) - \sec^{-1}2\right)$.

Sol. (a) Let $y = \cot^{-1}\sqrt{\frac{1+\cos 2x}{1-\cos 2x}}$

$\Rightarrow y = \cot^{-1}\sqrt{\frac{2\cos^2 x}{2\sin^2 x}} = \cot^{-1}\left(\left|\frac{\cos x}{\sin x}\right|\right) = \cot^{-1}|\cot x| = \cot^{-1}\cot x$

As $\cot^{-1}\cot x = x$, when $x \in (0, \pi)$.

So, $y = x$.

$\because x \in \left(0, \frac{\pi}{2}\right)$, $\cot x > 0 \ \therefore |\cot x| = \cot x$.

OR

(b) $\sin\left(\tan^{-1}(-\sqrt{3})-\sec^{-1}2\right)=\sin\left(-\frac{\pi}{3}-\frac{\pi}{3}\right)=\sin\left(-\frac{2\pi}{3}\right)=\sin\left(-\pi+\frac{\pi}{3}\right)=-\sin\frac{\pi}{3}=-\frac{\sqrt{3}}{2}$.

27. (a) Find the general solution of the differential equation $y^2dx+(x^2-xy+y^2)dy=0$.

OR

(b) Find the particular solution of the differential equation $\frac{dy}{dx}=y\tan x$, given that $y=2$ if $x=0$.

Sol. (a) Rewriting the D.E., $\frac{dx}{dy}+\frac{x^2-xy+y^2}{y^2}=0$

$\Rightarrow \frac{dx}{dy}+\frac{x^2}{y^2}-\frac{xy}{y^2}+\frac{y^2}{y^2}=0$

$\Rightarrow \frac{dx}{dy}+\left(\frac{x}{y}\right)^2-\frac{x}{y}+1=0$

Put $x=vy\Rightarrow \frac{dx}{dy}=v+y\frac{dv}{dy}$.

$\therefore v+y\frac{dv}{dy}+v^2-v+1=0$

$\Rightarrow y\frac{dv}{dy}+v^2+1=0$

$\Rightarrow \frac{dv}{v^2+1}+\frac{dy}{y}=0$

$\Rightarrow \int\frac{dv}{v^2+1}+\int\frac{dy}{y}=0$

$\Rightarrow \tan^{-1}v+\log|y|=C$

$\Rightarrow \tan^{-1}\frac{x}{y}+\log|y|=C$ or $x=y\tan\left[C-\log|y|\right]$.

OR

(b) Rewriting the D.E., we get $\int\frac{dy}{y}=\int\tan x\,dx$

$\Rightarrow \log|y|=-\log|\cos x|+\log|C|$

$\Rightarrow \log|y|=\log\left|\frac{C}{\cos x}\right|$

$\Rightarrow y=\frac{C}{\cos x}$

It is given that $y=2$ if $x=0$.

So, $2=\frac{C}{\cos 0}\Rightarrow C=2\times1=2$.

Hence, the required particular solution is $y=\frac{2}{\cos x}$ or $y=2\sec x$.

30. Find $\int_0^1 x\sin^{-1}x\,dx$.

Sol. Consider $\int x\sin^{-1}x\,dx = \sin^{-1}x\int x\,dx - \int\left[\frac{d}{dx}(\sin^{-1}x)\int x\,dx\right]dx$

$$= \frac{x^2}{2}\sin^{-1}x - \frac{1}{2}\int\frac{x^2}{\sqrt{1-x^2}}dx$$

$$= \frac{x^2}{2}\sin^{-1}x + \frac{1}{2}\int\frac{(1-x^2)-1}{\sqrt{1-x^2}}dx$$

$$= \frac{x^2}{2}\sin^{-1}x + \frac{1}{2}\int\left[\sqrt{1-x^2} - \frac{1}{\sqrt{1-x^2}}\right]dx$$

$$= \frac{x^2}{2}\sin^{-1}x + \frac{1}{2}\left[\left(\frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}x\right) - \sin^{-1}x\right]$$

$$= \frac{x^2}{2}\sin^{-1}x + \frac{x}{4}\sqrt{1-x^2} - \frac{1}{4}\sin^{-1}x$$

$$\therefore \int_0^1 x\sin^{-1}x\,dx = \left[\frac{x^2}{2}\sin^{-1}x + \frac{x}{4}\sqrt{1-x^2} - \frac{1}{4}\sin^{-1}x\right]_0^1$$

$$= \left[\frac{1}{2}\sin^{-1}1 + 0 - \frac{1}{4}\sin^{-1}1\right] - \left[0 + 0 - \frac{1}{4}\sin^{-1}0\right]$$

$$= \left[\frac{1}{2}\times\frac{\pi}{2} - \frac{1}{4}\times\frac{\pi}{2}\right]$$

$$= \frac{\pi}{8}.$$

32. If $x = 3\sin t - \sin 3t$, $y = 3\cos t - \cos 3t$, find $\frac{dy}{dx}$ and prove that $\frac{d^2y}{dx^2} = \frac{-\text{cosec}^3 2t.\text{cosec}\,t}{3}$.

Sol. $x = 3\sin t - \sin 3t$, $y = 3\cos t - \cos 3t$

$$\Rightarrow \frac{dx}{dt} = 3\cos t - 3\cos 3t,\ \frac{dy}{dt} = -3\sin t + \sin 3t\times 3$$

$$\Rightarrow \frac{dx}{dt} = 3(\cos t - \cos 3t),\ \frac{dy}{dt} = 3(\sin 3t - \sin t)$$

$$\Rightarrow \frac{dx}{dt} = 3\left[-2\sin\frac{t+3t}{2}\sin\frac{t-3t}{2}\right],\ \frac{dy}{dt} = 3\left[2\cos\frac{3t+t}{2}\sin\frac{3t-t}{2}\right]$$

$$\Rightarrow \frac{dx}{dt} = 3[-2\sin 2t\sin(-t)],\ \frac{dy}{dt} = 3[2\cos 2t\sin t]$$

$$\Rightarrow \frac{dx}{dt} = 6\sin 2t\sin t,\ \frac{dy}{dt} = 6\cos 2t\sin t$$

$$\because \frac{dy}{dx} = \frac{dy}{dt}\times\frac{dt}{dx}$$

$$\therefore \frac{dy}{dx} = \frac{6\cos 2t\sin t}{6\sin 2t\sin t}$$

$$\Rightarrow \frac{dy}{dx} = \cot 2t$$

Differentiating w.r.t. x both the sides, $\frac{d^2y}{dx^2} = -\text{cosec}^2 2t\times 2\frac{dt}{dx}$

$$\Rightarrow \frac{d^2y}{dx^2} = -\text{cosec}^2 2t \times 2 \times \frac{1}{6\sin 2t \sin t}$$

$$\therefore \frac{d^2y}{dx^2} = \frac{-\text{cosec}^3 2t.\text{cosec}\, t}{3}.$$

33. Prove that the line through points A(0, –1, –1) and B(4, 5, 1) intersects the line through points C(3, 9, 4) and D(–4, 4, 4). Hence, write the equation of line passing through the point of intersection of lines AB and CD as well as origin.

Sol. Equation of line AB : $\frac{x-0}{4-0} = \frac{y+1}{5+1} = \frac{z+1}{2}$ i.e., $\frac{x}{2} = \frac{y+1}{3} = \frac{z+1}{1} = \lambda$.

Equation of line CD : $\frac{x-3}{-4-3} = \frac{y-9}{4-9} = \frac{z-4}{4-4}$ i.e., $\frac{x-3}{7} = \frac{y-9}{5} = \frac{z-4}{0} = \mu$.

Taking the coordinates of random points on the lines AB and CD as $P(2\lambda, 3\lambda-1, \lambda-1)$ and $Q(7\mu+3, 5\mu+9, 4)$.

If AB and CD intersects, then points P and Q must coincide i.e., their coordinates must be same for some values of λ and μ.

That is, $2\lambda = 7\mu+3$ ...(i), $3\lambda-1 = 5\mu+9$ ...(ii), $\lambda-1 = 4$ ...(iii)

Solving (i) and (iii), we get $\lambda = 5$, $\mu = 1$.

Now we shall check if $\lambda = 5$, $\mu = 1$ satisfy (ii) or not.

LHS of (ii): $3\lambda-1 = 3(5)-1 = 14$, RHS of (ii): $5\mu+9 = 5(1)+9 = 14$.

Clearly, the values of $\lambda = 5$, $\mu = 1$ satisfy (ii).

Hence, lines AB and CD intersect each other.

And the point of intersection will be P(10, 14, 4).

Let O(0, 0, 0).

Then the equation of line OP is $\frac{x-0}{10-0} = \frac{y-0}{14-0} = \frac{z-0}{4}$ i.e., $\frac{x}{5} = \frac{y}{7} = \frac{z}{2}$.

☑ Q.P. Code - 65/1/3

02. The length of perpendicular drawn from the point (3, 4, 2) on the line $\frac{x}{0} = \frac{y}{0} = \frac{z}{1}$ is

(a) 2 (b) 9 (c) 5 (d) $\sqrt{29}$

Sol. (c) In 3-D geometry, a line through (0, 0, 0) and with direction ratios 0, 0, 1 is simply the z-axis.

That is, $\frac{x}{0} = \frac{y}{0} = \frac{z}{1}$ represents the z-axis.

Now foot of perpendicular drawn from (3, 4, 2) to the z-axis is (0, 0, 2).

Therefore, we just need to find the distance between these two points, using distance formula.

Hence, required length of perpendicular $= \sqrt{3^2+4^2+0^2} = \sqrt{9+16} = \sqrt{25} = 5$.

05. If $\sin^{-1} x + \pi = y$, then

(a) $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$ (b) $-\frac{3\pi}{2} \le y \le -\frac{\pi}{2}$ (c) $\frac{\pi}{2} \le y \le \frac{3\pi}{2}$ (d) $0 \le y \le \pi$

Sol. (c) As $-\frac{\pi}{2} \le \sin^{-1} x \le \frac{\pi}{2}$ $\Rightarrow -\frac{\pi}{2}+\pi \le \sin^{-1} x + \pi \le \frac{\pi}{2}+\pi$ $\therefore \frac{\pi}{2} \le y \le \frac{3\pi}{2}$.

06. If $A = \left[a_{ij}\right]_{3\times 3}$ is a scalar matrix, then which of the following must be true?

(a) A must be a symmetric matrix (b) A must be a skew-symmetric matrix

(c) A must be an identity matrix (d) A must be a null matrix

Sol. (a) The scalar matrix A must be a symmetric matrix.

10. If A and B are symmetric matrices of same order, then $(AB - BA)$ is

(a) zero matrix (b) identity matrix (c) symmetric matrix (d) skew symmetric matrix

Sol. (d) Let $P = AB - BA$

$\Rightarrow P^T = (AB - BA)^T = (AB)^T - (BA)^T = B^T A^T - A^T B^T$

$\Rightarrow P^T = BA - AB = -P$ $[\because A^T = A,\ B^T = B$

Thus, $P = AB - BA$ is a skew-symmetric matrix.

11. The least value of $f(x) = e^{-x}$ in $[0, 3]$ is

(a) e^{-3} (b) -1 (c) 1 (d) $-e^3$

Sol. (a) $\because f'(x) = -e^{-x} < 0$ for all $x \in \mathbb{R}$ i.e., f is a strictly decreasing function.

Since the function is strictly decreasing over the interval [0, 3], its maximum value will occur at the smallest x, and its **least value** will occur at the largest x.

Therefore, the least value of $f(x)$ is $f(3) = e^{-3}$.

16. The order and degree of the differential equation

$\frac{d}{dx}(y')^3 + (y')^3 = 1$ respectively are, where $y' = \frac{dy}{dx}$

(a) 1, 3 (b) 2, 1 (c) 3, 1 (d) 3, 2

Sol. (b) Since $\frac{d}{dx}(y')^3 + (y')^3 = 1$ can be rewritten as $\frac{d}{dx}\left(\frac{dy}{dx}\right)^3 + \left(\frac{dy}{dx}\right)^3 = 1$

$\Rightarrow 3\left(\frac{dy}{dx}\right)^2 \times \frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 = 1$

Clearly, order : 2 and degree : 1.

23. (a) Simplify : $\sin^{-1}\sqrt{\frac{1+\cos 2x}{2}},\ 0 < x < \frac{\pi}{2}$.

OR

(b) Evaluate : $\cos\left[\sin^{-1}(-1) - \tan^{-1}(-\sqrt{3})\right]$.

Sol. (a) Let $y = \sin^{-1}\sqrt{\frac{1+\cos 2x}{2}}$

$\Rightarrow y = \sin^{-1}\sqrt{\frac{2\cos^2 x}{2}} = \sin^{-1}\sqrt{\cos^2 x} = \sin^{-1}|\cos x| = \sin^{-1}\cos x$ $\left[\because 0 < x < \frac{\pi}{2},\ \cos x > 0\right.$

$\Rightarrow y = \sin^{-1}\sin\left(\frac{\pi}{2} - x\right)$

$\therefore y = \left(\frac{\pi}{2} - x\right)$.

As $0 < x < \frac{\pi}{2}$ $\Rightarrow 0 - \frac{\pi}{2} < x - \frac{\pi}{2} < \frac{\pi}{2} - \frac{\pi}{2}$ $\Rightarrow -\frac{\pi}{2} < x - \frac{\pi}{2} < 0$ $\Rightarrow 0 < \frac{\pi}{2} - x < \frac{\pi}{2}$.

Also $\sin^{-1}\sin x = x$ when $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

OR

(b) $\cos\left[\sin^{-1}(-1) - \tan^{-1}(-\sqrt{3})\right] = \cos\left[-\frac{\pi}{2} - \left(-\frac{\pi}{3}\right)\right] = \cos\left[-\frac{\pi}{2} + \frac{\pi}{3}\right] = \cos\left(-\frac{\pi}{6}\right) = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$.

24. Using vectors, find the area of ΔABC with vertices A(1, 2, 3), B(2, –1,4) and C(4, 5, –1).

Sol. $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = (2\hat{i} - \hat{j} + 4\hat{k}) - (\hat{i} + 2\hat{j} + 3\hat{k}) = \hat{i} - 3\hat{j} + \hat{k}$,

$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = (4\hat{i} + 5\hat{j} - \hat{k}) - (\hat{i} + 2\hat{j} + 3\hat{k}) = 3\hat{i} + 3\hat{j} - 4\hat{k}$.

$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 1 \\ 3 & 3 & -4 \end{vmatrix} = 9\hat{i} + 7\hat{j} + 12\hat{k}$$

Now area of $\Delta ABC = \frac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{AC}\right|$

$$= \frac{1}{2}\sqrt{81 + 49 + 144} = \frac{\sqrt{274}}{2} \text{ Sq. units.}$$

26. Find $\int_0^1 \log(1 + x^2)dx$.

Sol. Consider $\int \log(1 + x^2)dx = \log(1 + x^2)\int 1\,dx - \int\left[\frac{d}{dx}\left(\log(1 + x^2)\right)\int 1\,dx\right]dx$

$$= x\log(1 + x^2) - \int\left[\frac{2x}{1 + x^2} \times x\right]dx$$

$$= x\log(1 + x^2) - 2\int\left[\frac{x^2}{1 + x^2}\right]dx = x\log(1 + x^2) - 2\int\left[1 - \frac{1}{1 + x^2}\right]dx$$

$$= x\log(1 + x^2) - 2\left[x - \tan^{-1}x\right]$$

Now $\int_0^1 \log(1 + x^2)dx = \left[x\log(1 + x^2) - 2\left(x - \tan^{-1}x\right)\right]_0^1$

$$= \left[1 \times \log(1 + 1^2) - 2\left(1 - \tan^{-1}1\right)\right] - \left[0\log(1 + 0^2) - 2\left(0 - \tan^{-1}0\right)\right]$$

$$= \left[\log 2 - 2\left(1 - \frac{\pi}{4}\right)\right] - 0$$

$$= \log 2 + \frac{\pi}{2} - 2.$$

30. (a) Find the general solution of the differential equation $(y^2 - x^2)dx = 2xydy$.

OR

(b) Find the particular solution of the differential equation $(1 + e^{2x})dy + (1 + y^2)e^x dx = 0$, given that $y(1) = 0$.

Sol. (a) Rewriting the D.E., $\frac{dy}{dx} = \frac{(y^2 - x^2)}{2xy}$ i.e., $\frac{dy}{dx} = \frac{1}{2}\left[\frac{y}{x} - \frac{x}{y}\right]$

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$.

So, $v + x\frac{dv}{dx} = \frac{1}{2}\left[v - \frac{1}{v}\right]$

$\Rightarrow 2v + 2x\frac{dv}{dx} = v - \frac{1}{v}$

$\Rightarrow 2x\frac{dv}{dx} = -v - \frac{1}{v} \quad \Rightarrow 2x\frac{dv}{dx} = \frac{-v^2 - 1}{v}$

$$\Rightarrow \int \frac{2v}{v^2+1}dv = \int -\frac{dx}{x} \qquad \left[\because \int \frac{f'(x)}{f(x)}dx = \log|f(x)| + C\right.$$

$$\Rightarrow \log|v^2+1| = -\log|x| + \log|C|$$

$$\Rightarrow \log\left|\frac{y^2}{x^2}+1\right| = \log\left|\frac{C}{x}\right|$$

$$\Rightarrow \frac{y^2+x^2}{x^2} = \frac{C}{x} \text{ or } y^2+x^2 = xC.$$

OR

(b) Rewriting the D.E., we get $\int \frac{dy}{1+y^2} + \int \frac{e^x\,dx}{1+e^{2x}} = 0$

$$\Rightarrow \int \frac{dy}{1+y^2} + \int \frac{e^x\,dx}{1+(e^x)^2} = 0 \qquad \left[\text{Put } e^x = t \Rightarrow e^x dx = dt \text{ in second integral}\right.$$

$$\Rightarrow \int \frac{dy}{1+y^2} + \int \frac{dt}{1+t^2} = 0$$

$$\Rightarrow \tan^{-1} y + \tan^{-1} t = C$$

$$\Rightarrow \tan^{-1} y + \tan^{-1} e^x = C$$

It is given that $y(1) = 0$.

So, $\tan^{-1} 0 + \tan^{-1} e^1 = C \qquad \Rightarrow C = \tan^{-1} e$

Therefore, the required particular solution is $\tan^{-1} y + \tan^{-1} e^x = \tan^{-1} e$.

Also $\tan^{-1} y + \tan^{-1} e^x = \tan^{-1} e$ can be simplified as $\tan\left[\tan^{-1} y + \tan^{-1} e^x\right] = e$

$$\Rightarrow \frac{\tan\left[\tan^{-1} y\right] + \tan\left[\tan^{-1} e^x\right]}{1 - \tan\left[\tan^{-1} y\right]\tan\left[\tan^{-1} e^x\right]} = e \text{ i.e., } \frac{y+e^x}{1-ye^x} = e \text{ i.e., } y + e^x = e(1-ye^x).$$

32. Show that line AB passing through points A(0, 4, 1), B(2, 3, –1) and the line CD passing through points C(4, 5, 0), D(2, 6, 2) are parallel. Also, find distance between them.

Sol. Equation of line AB : $\frac{x-0}{2-0} = \frac{y-4}{3-4} = \frac{z-1}{-1-1}$ i.e., $\frac{x}{2} = \frac{y-4}{-1} = \frac{z-1}{-2}$.

Equation of line CD : $\frac{x-4}{2-4} = \frac{y-5}{6-5} = \frac{z-0}{2-0}$ i.e., $\frac{x-4}{-2} = \frac{y-5}{1} = \frac{z}{2}$.

The direction ratios of line AB are 2, –1, –2 and that of line CD are –2, 1, 2.

Since $\frac{2}{-2} = \frac{-1}{1} = \frac{-2}{2}$ i.e., the d.r.'s are proportional.

Hence, AB || CD.

Now AB : $\vec{r} = 4\hat{j} + \hat{k} + \lambda(2\hat{i} - \hat{j} - 2\hat{k})$ and CD : $\vec{r} = 4\hat{i} + 5\hat{j} + \lambda(-2\hat{i} + \hat{j} + 2\hat{k})$.

Clearly, $\vec{a}_1 = 4\hat{j} + \hat{k}$, $\vec{b}_1 = 2\hat{i} - \hat{j} - 2\hat{k}$; $\vec{a}_2 = 4\hat{i} + 5\hat{j}$, $\vec{b}_2 = -2\hat{i} + \hat{j} + 2\hat{k}$.

Then $\vec{a}_2 - \vec{a}_1 = (4\hat{i} + 5\hat{j}) - (4\hat{j} + \hat{k}) = 4\hat{i} + \hat{j} - \hat{k}$.

Also we shall take $\vec{b}_1 = \vec{b}$.

Taking $(\vec{a}_2 - \vec{a}_1) \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 1 & -1 \\ 2 & -1 & -2 \end{vmatrix} = -3\hat{i} + 6\hat{j} - 6\hat{k}$.

Hence, the required distance between the lines AB and CD $= \dfrac{\left|(\vec{a}_2 - \vec{a}_1) \times \vec{b}\right|}{\left|\vec{b}\right|}$

$$= \frac{\left|-3\hat{i}+6\hat{j}-6\hat{k}\right|}{\left|2\hat{i}-\hat{j}-2\hat{k}\right|} = \frac{\sqrt{9+36+36}}{\sqrt{4+1+4}} = \frac{\sqrt{81}}{\sqrt{9}} = \frac{9}{3} = 3 \text{ units}.$$

35. If $x = a(\sin t - t\cos t)$ and $y = b(\cos t + t\sin t)$, then find $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$.

Sol. $y = b(\cos t + t\sin t)$, $x = a(\sin t - t\cos t)$

Differentiating w.r.t. t, we get $\dfrac{dy}{dt} = b(-\sin t + t\cos t + \sin t.1)$, $\dfrac{dx}{dt} = a(\cos t + t\sin t - \cos t.1)$

$\Rightarrow \dfrac{dy}{dt} = b\,t\cos t$, $\dfrac{dx}{dt} = a\,t\sin t$

$\because \dfrac{dy}{dx} = \left(\dfrac{dy}{dt}\right)\left(\dfrac{dt}{dx}\right)$ $\quad \therefore \dfrac{dy}{dx} = (b\,t\cos t)\left(\dfrac{1}{a\,t\sin t}\right)$

$\Rightarrow \dfrac{dy}{dx} = \left(\dfrac{b}{a}\right)\cot t$

Now differentiating w.r.t. x, we get $\dfrac{d^2y}{dx^2} = -\dfrac{b}{a}\operatorname{cosec}^2 t \times \dfrac{dt}{dx}$

$\Rightarrow \dfrac{d^2y}{dx^2} = -\dfrac{b}{a}\operatorname{cosec}^2 t \times \dfrac{1}{a\,t\sin t}$

$\therefore \dfrac{d^2y}{dx^2} = -\dfrac{b\operatorname{cosec}^3 t}{a^2 t}$.

With regards,
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✔ Quality Mathematics Resources
✔ Important Practice Questions & Updates
✔ Board Exam Discussions & Solutions
✔ Healthy Academic Interaction

ⓘ How to Join?

Please **scan the QR-Code** corresponding to your category (Teachers / Class IX - X Students / Class XI - XII Students) to join the relevant group.
Alternatively, you can **touch the QR-Code** too, after opening in the Drive PDF App.

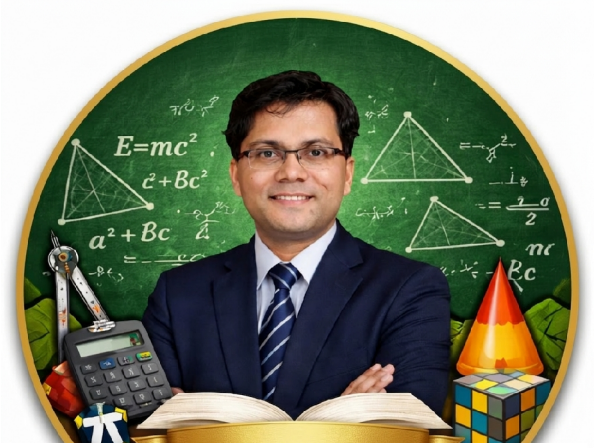
✪ Important Guidelines

- Teachers are requested NOT to join student groups.
- Students are requested NOT to join teachers' groups.

☑ If you are already a member of any of our existing groups, please avoid joining another group to prevent repeated notifications of the same resources.
Instead, you may share this opportunity with your colleagues or students who may benefit from these Mathematics learning communities.

With Regards
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Dedicated to helping students and teachers strengthen conceptual understanding and excel in Mathematics.